CALCULATION EXAMPLES

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MATRIX DETERMINANT

The determinant of a matrix is a number that can be calculated based on the matrix element values. For matrix M the determinant may be indicated by det(M). The following example illustrates the calculation of a matrix determinant if the number of rows and columns in the matrix is two (Sydsæter-Hammond (2008), page 573):

$$\det\begin{pmatrix}1 & 3\\ 1 & 5\end{pmatrix} = 1\cdot 5 - 3\cdot 1 = 2$$

Some matrix calculation results may be calculated with R (R Core Team (2019). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/). In R the determinant of a matrix may be calculated. For example in case of the example matrix determinant could be calculated as follows:

```
matrix2=matrix(nrow=2,ncol=2)

matrix2[1,1]=1
matrix2[1,2]=3
matrix2[2,1]=1
matrix2[2,2]=5

matrix2

det(matrix2)
```

If the number of rows and columns is more than two, then the calculation of the matrix determinant is more complex than in the previous example, as the following example suggests (Sydsæter-Hammond (2008), page 577):

$$\det\begin{pmatrix}1 & 5 & 0\\ 1 & 5 & 0\\ 0 & 0 & 3\end{pmatrix} = 1\cdot(5\cdot 3 - 0\cdot 0) - 5\cdot(1\cdot 3 - 0\cdot 0) + 0\cdot(1\cdot 0 - 5\cdot 0) = 0$$

The matrix determinant can also be calculated in R.

```
matrix3=matrix(nrow=3,ncol=3)

matrix3[1,1]=1
matrix3[1,2]=5
matrix3[1,3]=0
matrix3[2,1]=1
matrix3[2,2]=5
matrix3[2,3]=0
matrix3[3,1]=0
matrix3[3,2]=0
matrix3[3,3]=3
```

```
matrix3

det(matrix3)
```

EIGENVALUE CALCULATION

Assume in the following that matrix M is symmetric. According to the spectral decomposition theorem a diagonal matrix D and a matrix C exist so that (Medvegyev (2002), page 454):

$$D = C^T \cdot M \cdot C$$

The diagonal values of matrix D are usually referred to as eigenvalues.

Theoretically the correlation matrix is a symmetric matrix, for example if the number of columns in the correlation matrix is two, then the eigenvalues of the correlation matrix $\begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}$ are 1.9 and 0.1.

$$\begin{pmatrix} 1.9 & 0 \\ 0 & 0.1 \end{pmatrix} = \begin{pmatrix} 0.7071 & 0.7071 \\ -0.669 & 0.7432 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0.7071 & -0.669 \\ 0.7071 & 0.7432 \end{pmatrix}$$

The eigenvalues (and the eigenvectors) may also be calculated in R.

```
matrix2=matrix(nrow=2,ncol=2)

matrix2[1,1]=1
matrix2[1,2]=0.9
matrix2[2,1]=0.9
matrix2[2,2]=1

matrix2

eigen(matrix2)
eigen(matrix2)$values
```

It may also be illustrated with R calculations that the diagonal matrix (in which the diagonal values are the eigenvalues) may be calculated based on the matrix related to the eigenvectors and the correlation matrix in the example (the transpose of matrix M can be calculated as t(M) in R).

```
t(eigen(matrix2)$vectors)%*%matrix2%*% eigen(matrix2)$vectors
```

References

Medvegyev, P. (2002): Valószínűségszámítás, Fejezetek a matematikai analízisből és a valószínűségszámításból (in Hungarian). Aula

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Sydsæter, K. – Hammond, P. (2008): Essential mathematics for economic analysis. Third edition. Prentice-Hall, Inc.